4.2 Quadratic Relations

The Galleria, in BCE Place in Toronto, has many arches. These curved structures are used to span a space while supporting weight. Just as a linear relation can be modelled with a linear equation, some non-linear relations, such as the shape of an arch, can be modelled using non-linear equations.

Tools

- grid paper

quadratic relation

- a relation whose equation is in the form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$

parabola

- the graph of a quadratic relation, which is U-shaped and symmetrical

vertex

- the point on a parabola where the curve changes direction
- the maximum point if the parabola opens down
- the minimum point if the parabola opens up

axis of symmetry

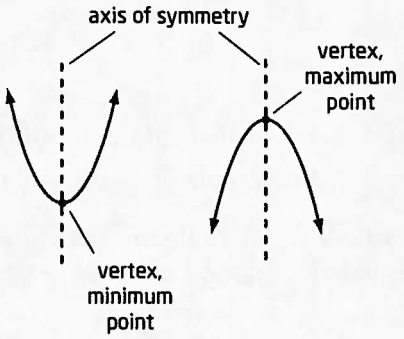
- the line that divides a figure into two congruent parts

Investigate A

How can you compare relations of the form $y = ax^2 + bx + c$?

1. Make a table of values for each relation, using integer values of x from -3 to $+3$.
   a) $y = x^2$ b) $y = 2x^2$ c) $y = x^2 + 2x + 3$
   d) $y = -x^2$ e) $y = -0.5x^2 + 3$
2. Graph all the relations in step 1 on the same set of axes. Plot each set of ordered pairs and draw a smooth curve through the points.
3. Reflect Describe the graphs you created in as many ways as you can. What is similar about the graphs? What is different?

The relation described by $y = ax^2 + bx + c$ is called a **quadratic relation**. The graph of a quadratic relation is called a **parabola**. A parabola has a minimum point or a maximum point called the **vertex**. It is also symmetrical about a vertical line drawn through the vertex, called the **axis of symmetry**.



Investigate B

How can you use finite differences to determine if a relation is linear or quadratic?

finite differences

- differences found from the y-values in tables with evenly spaced x-values
- first differences are the differences between consecutive y-values, second differences are the differences between consecutive first differences, and so on

1. Copy and complete the table for each linear relation. Calculate the y-values. Then, calculate the first differences by subtracting consecutive y-values.

a) $y = 2x - 5$

x	y	First Differences
-2	-9	
-1	-7	-7 - (-9) = 2
0		
1		
2		

b) $y = -6x + 2$

x	y	First Differences
-2	14	
-1	8	8 - 14 = -6
0		
1		
2		

2. What is true about the first differences for a linear relation?

3. Copy and complete the table for each quadratic relation. Calculate the y-values and first differences. Then, calculate the second differences by subtracting successive first differences.

a) $y = x^2 - 4$

x	y	First Differences	Second Differences
-2	0		
-1	-3	-3 - 0 = -3	-1 - (-3) = 2
0	-4	-4 - (-3) = -1	
1			
2			

b) $y = 2x^2 + 3x - 1$

x	y	First Differences	Second Differences
-2	1		
-1	-2		
0			
1			
2			

4. **a)** What is true about the first differences for a quadratic relation?
   b) What is true about the second differences for a quadratic relation?

5. Reflect Write a rule for using finite differences to determine whether a relation is linear or quadratic.

Example Galleria Arches

Each arch in the BCE Place Galleria can be approximated by the relation $y = -0.55x^2 + 26$, where y is the height, in metres, above the floor and x is the width, in metres, from the centre of the hallway.

a) Graph the quadratic relation.

b) Describe the shape of the arch.

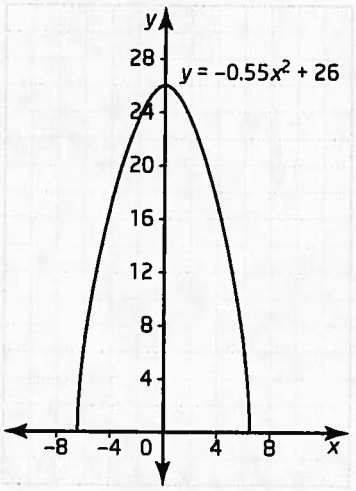
c) How tall and wide is the arch?

Solution

a) Method 1: Use Pencil and Paper

Use a table of values to help you sketch the graph.

x	y
-6	6.2
-4	17.2
-2	23.8
0	26.0
2	23.8
4	17.2
6	6.2

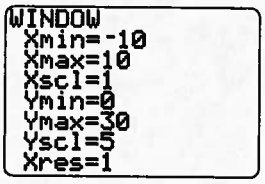


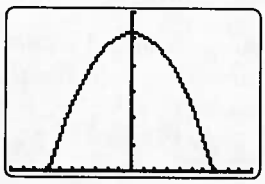
Method 2: Use a Graphing Calculator

Enter the equation using [Y=].

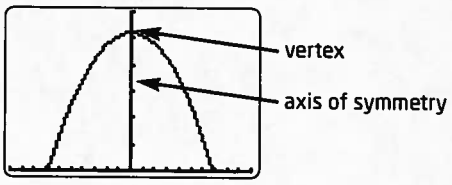
Press [WINDOW] and enter the settings shown.

Then, press [GRAPH].

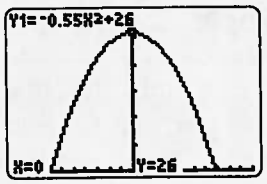




b) You can see that the shape of the arch is parabolic. The parabola is symmetrical about a vertical line, the y-axis. The graph has a maximum point.



c) You can read the maximum y-value from the pencil-and-paper graph or use the TRACE feature on the graphing calculator.



I can see that the maximum occurs when $x = 0$. From the equation, when $x = 0$, $y = 26$.

Since the maximum value of y is 26, the height of each arch is 26 m.

The x-axis represents the floor of the hallway. The width of each arch is the difference between the two x-intercepts. From the pencil-and-paper graph, the x-intercepts appear to be about 7 and -7. Use the TRACE feature on the graphing calculator to find that the curve crosses the x-axis at about -6.9 and $+6.9$.

$6.9 - (-6.9) = 13.8$

The width of each arch is about 13.8 m.

Technology Tip

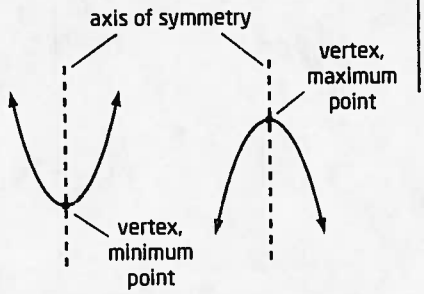
You can get a better approximation of the x-intercepts by zooming in.

- Position the cursor near one of the x-intercepts.
- Press ZOOM, select **2:Zoom In**, and then press ENTER.
- Press TRACE and reposition the cursor.

The accuracy of the approximation improves each time you repeat these steps.

Key Concepts

- The relation defined by $y = ax^2 + bx + c$ is a quadratic relation.
- The graph of a quadratic relation is called a parabola.
- The vertex of a parabola is either the minimum point or the maximum point on the graph.



- A parabola is symmetric about a vertical line that passes through the vertex. This line is the axis of symmetry.
- If a relation is quadratic, the second differences are constant, but the first differences are not.

Communicate Your Understanding

C1 El-Noor used the following incorrect technique to determine that the relation is not quadratic. Explain the flaw in his reasoning.

x	y	First Differences	Second Differences
−3	13		
		−10	
−2	3		2
		−8	
0	−5		10
		2	
1	−3		4
		6	
2	3		18
		24	
4	27		

C2 In Section 4.1, Investigate Part A, you found that the relationship between thumb length and palm area is non-linear. Is the relation quadratic? Explain.